UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 4TH FEBRUARY 2016

Organised by the **United Kingdom Mathematics Trust**
and supported by

Institute and Faculty of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

http://www.ukmt.org.uk

1. What is the value of $6102 - 2016$?

   A 3994 B 4086 C 4096 D 4114 E 4994

2. Which of the following fractions is closest to 1?

   A $\frac{7}{8}$ B $\frac{8}{7}$ C $\frac{9}{10}$ D $\frac{10}{11}$ E $\frac{11}{10}$

3. How many of these five expressions give answers which are *not* prime numbers?

   $1^2 + 2^2$ $2^2 + 3^2$ $3^2 + 4^2$ $4^2 + 5^2$ $5^2 + 6^2$

   A 0 B 1 C 2 D 3 E 4

4. Amrita is baking a cake today. She bakes a cake every fifth day. How many days will it be before she next bakes a cake on a Thursday?

   A 5 B 7 C 14 D 25 E 35

5. When travelling from London to Edinburgh by train, you pass a sign saying 'Edinburgh 200 miles'. Then, $3\frac{1}{2}$ miles later, you pass another sign saying 'Half way between London and Edinburgh'.

   How many miles is it by train from London to Edinburgh?

   A 393 B $396\frac{1}{2}$ C 400 D $403\frac{1}{2}$ E 407

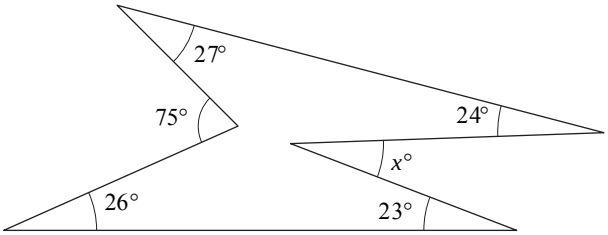
6. One third of the animals in Jacob's flock are goats, the rest are sheep. There are twelve more sheep than goats.

   How many animals are there altogether in Jacob's flock?

   A 12 B 24 C 36 D 48 E 60

7. In the diagram, what is the value of x?

   A 23 B 24 C 25 D 26 E 27



8. What is the value of $2.017 \times 2016 - 10.16 \times 201.7$?

   A 2.016 B 2.017 C 20.16 D 2016 E 2017

9. The world's fastest tortoise is acknowledged to be a leopard tortoise from County Durham called Bertie. In July 2014, Bertie sprinted along a 5.5 m long track in an astonishing 19.6 seconds.

   What was Bertie's approximate average speed in km per hour?

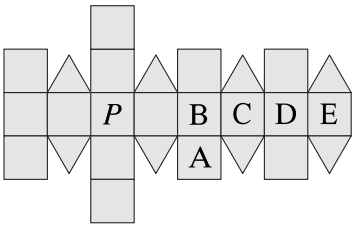
   A 0.1 B 0.5 C 1 D 5 E 10

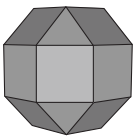
10. The angles of a quadrilateral taken in order are $x°$, $5x°$, $2x°$ and $4x°$. Which of the following is the quadrilateral?

    A kite B parallelogram C rhombus D arrowhead E trapezium

11. The net shown consists of squares and equilateral triangles. The net is folded to form a rhombicuboctahedron, as shown.

When the face marked *P* is placed face down on a table, which face will be facing up?





A B C D E

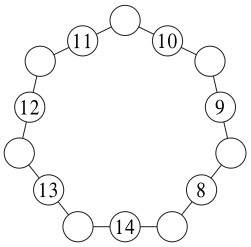
12. The sum of two numbers a and b is 7 and the difference between them is 2.

What is the value of $a \times b$?

A $8\frac{1}{4}$ B $9\frac{1}{4}$ C $10\frac{1}{4}$ D $11\frac{1}{4}$ E $12\frac{1}{4}$

13. The diagram shows a heptagon with a line of three circles on each side. Each circle is to contain exactly one number. The numbers 8 to 14 are distributed as shown and the numbers 1 to 7 are to be distributed to the remaining circles. The total of the numbers in each of the lines of three circles is to be the same.

What is this total?



A 18 B 19 C 20 D 21 E 22

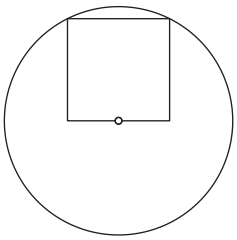
14. Tegwen has the same number of brothers as she has sisters. Each one of her brothers has 50% more sisters than brothers.

How many children are in Tegwen's family?

A 5 B 7 C 9 D 11 E 13

15. The circle has radius 1 cm. Two vertices of the square lie on the circle. One edge of the square goes through the centre of the circle, as shown.

What is the area of the square?



A $\frac{4}{5}$ cm^2 B $\frac{\pi}{5}$ cm^2 C 1 cm^2 D $\frac{\pi}{4}$ cm^2 E $\frac{5}{4}$ cm^2

---

16. How many of the following positive integers are divisible by 24?

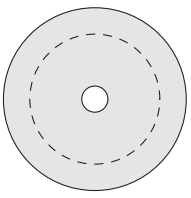
$2^2 \times 3^2 \times 5^2 \times 7^3$ $2^2 \times 3^2 \times 5^3 \times 7^2$ $2^2 \times 3^3 \times 5^2 \times 7^2$ $2^3 \times 3^2 \times 5^2 \times 7^2$

A 0 B 1 C 2 D 3 E 4

17. The shaded region in the diagram, bounded by two concentric circles, is called an *annulus*. The circles have radii 2 cm and 14 cm.

The dashed circle divides the area of this annulus into two equal areas.

What is its radius?



A 9 cm B 10 cm C 11 cm D 12 cm E 13 cm

18. The sum of the areas of the squares on the sides of a right-angled isosceles triangle is 72 cm^2.

What is the area of the triangle?

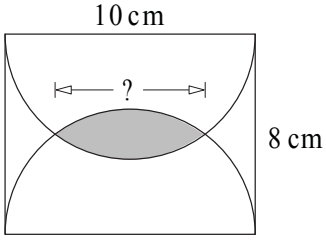
A 6 cm^2 B 8 cm^2 C 9 cm^2 D 12 cm^2 E 18 cm^2

19. A list of positive integers has a median of 8, a mode of 9 and a mean of 10.

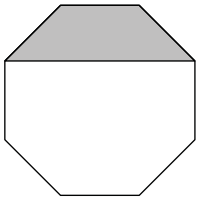
What is the smallest possible number of integers in the list?

A 5 B 6 C 7 D 8 E 9

20. Two semicircles are drawn in a rectangle as shown.
What is the width of the overlap of the two semicircles?
A 3 cm B 4 cm C 5 cm D 6 cm E 7 cm

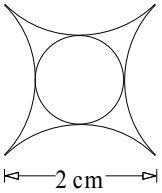


21. The diagram shows a regular octagon. What is the ratio of the area of the shaded trapezium to the area of the whole octagon?
A 1 : 4 B 5 : 16 C 1 : 3 D $\sqrt{2}$: 2 E 3 : 8



22. In a particular group of people, some always tell the truth, the rest always lie. There are 2016 in the group. One day, the group is sitting in a circle. Each person in the group says, "Both the person on my left and the person on my right are liars."
What is the difference between the largest and smallest number of people who could be telling the truth?
A 0 B 72 C 126 D 288 E 336

23. A Saxon silver penny, from the reign of Ethelbert II in the eighth century, was sold in 2014 for £78 000. A design on the coin depicts a circle surrounded by four equal arcs, each a quarter of a circle, as shown. The width of the design is 2 cm.
What is the radius of the small circle, in centimetres?
A $\frac{1}{2}$ B $2-\sqrt{2}$ C $\frac{1}{2}\sqrt{2}$ D $5-3\sqrt{2}$ E $2\sqrt{2}-2$



24. Every day, Aimee goes up an escalator on her journey to work. If she stands still, it takes her 60 seconds to travel from the bottom to the top. One day the escalator was broken so she had to walk up it. This took her 90 seconds.
How many seconds would it take her to travel up the escalator if she walked up at the same speed as before while it was working?
A 30 B 32 C 36 D 45 E 75

25. The tiling pattern shown uses two types of tile, regular hexagons and equilateral triangles, with the length of each side of the equilateral triangles equal to half the length of each side of the hexagons. A large number of tiles is used to cover a floor.
Which of the following is closest to the fraction of the floor that is shaded black?
A $\frac{1}{8}$ B $\frac{1}{10}$ C $\frac{1}{12}$ D $\frac{1}{13}$ E $\frac{1}{16}$